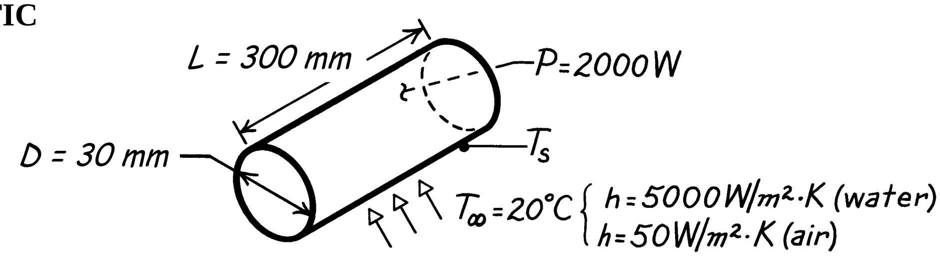
PROBLEM 1.17

KNOWN: Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

FIND: Thermal convection resistance and heater surface temperatures in water and air.

SCHEMATIC



ASSUMPTIONS: (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

ANALYSIS: With P = q_{conv}, Newton's law of cooling yields

$$P = hA\left(T_S - T_\infty\right) = h\pi DL\left(T_S - T_\infty\right)$$
$$T_S = T_\infty + \frac{P}{h\pi DL}.$$

From Eq. 1.11, the thermal resistance due to convection is given by

$$R_{t,conv} = \Delta T / q_x = \left(T_s - T_\infty\right) / P = 1 / h\pi DL$$

In water,

$$T_s = 20°C + \frac{2000\ W}{5000\ W/m^2 \cdot K \times \pi \times 0.03\ m \times 0.3\ m} = 34.2°C \qquad <$$

$$R_{t,conv} = 1/h\pi DL = 1/(5000\ W/m^2 \cdot K \times \pi \times 0.03\ m \times 0.3\ m) = 0.00707\ K/W \qquad <$$

In air,

$$T_s = 20°C + \frac{2000\ W}{50\ W/m^2 \cdot K \times \pi \times 0.03\ m \times 0.3\ m} = 1435°C \qquad <$$

$$R_{t,conv} = 1/h\pi DL = 1/(50\ W/m^2 \cdot K \times \pi \times 0.03\ m \times 0.3\ m) = 0.707\ K/W \qquad <$$

COMMENTS: (1) Air is much less effective than water as a heat transfer fluid. Hence, the cartridge temperature is much higher in air, so high, in fact, that the cartridge would melt. (2) In air, the high cartridge temperature would render radiation significant. (3) Larger thermal resistance corresponds to less effective heat transfer.